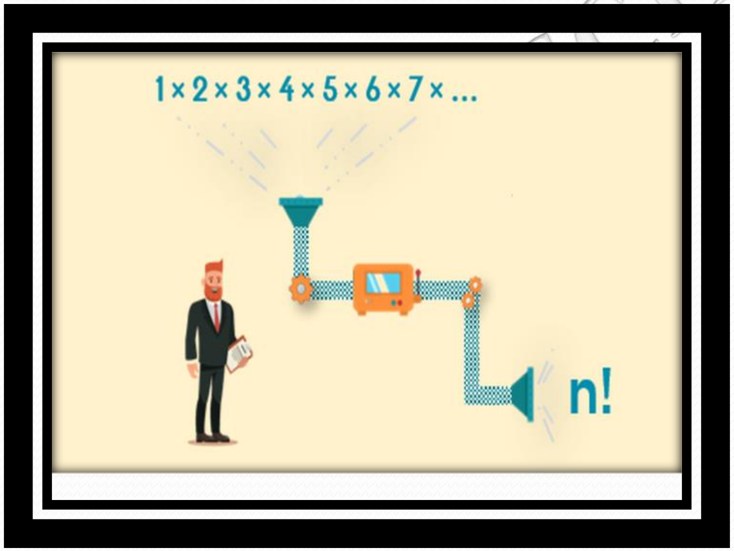
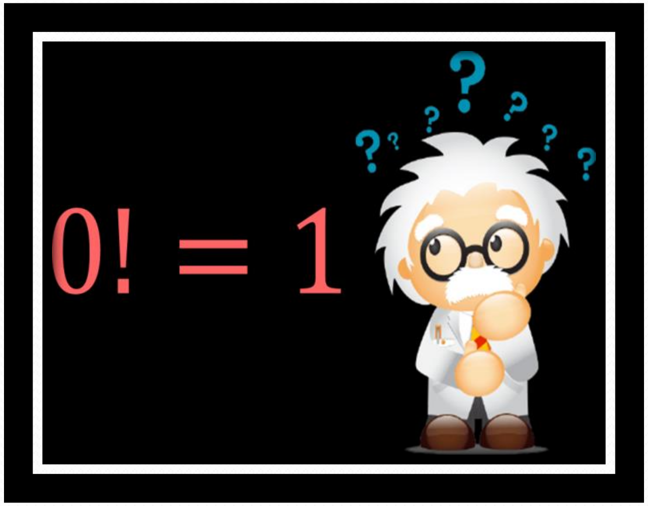
Permutation and Combination

Module-2





Recap

Fundamental Principle of Counting

"If an event can occur in **m** different ways , following which another event can occur in **n** different ways, then the total number of occurrence of the events in the given order is **m x n** ."

Generalised Form

Event 1: a_1 ways
Event 2: a_2 ways
.
.
Event n: a_n ways

Total number of ways is $a_1 \bullet a_2 \ldots \bullet a_n$

Factorial notation

- The notation ‘n!’ represents the product of first n natural numbers

$$1 \times 2 \times 3 \times \ldots\ldots\ldots\ldots\ldots\ldots (n-1)n = n!$$

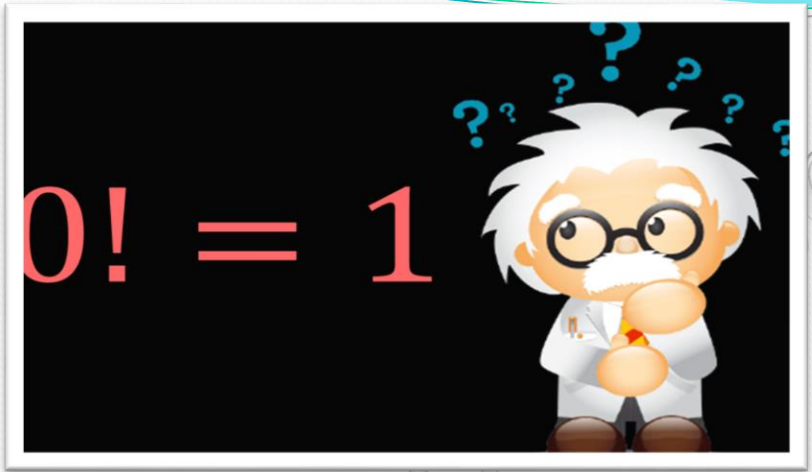
$1 = 1!$

$1 \times 2 = 2!$

$1 \times 2 \times 3 = 3!$

For a natural number ‘n’

$$\begin{aligned} n! &= n(n-1)! \\ &= n(n-1)(n-2)! \\ &= n(n-1)(n-2)(n-3)! \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$



$$n! = n\,(n-1)!$$

$$(n-1)! = \frac{n!}{n}$$

$$\text{For } n=1,\ 0! = \frac{1!}{1} = 1$$

Let's discuss:

Question 1:

Evaluate **(i)** $8!$ **(ii)** $4! - 3!$

Answer 1:

(i) $8! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320$

(ii) $4! = 1 \times 2 \times 3 \times 4 = 24$

$3! = 1 \times 2 \times 3 = 6$

$\therefore 4! - 3! = 24 - 6 = 18$

Question 2:

Is $3! + 4! = 7!$?

Answer 2:

$3! = 1 \times 2 \times 3 = 6$

$4! = 1 \times 2 \times 3 \times 4 = 24$

$\therefore 3! + 4! = 6 + 24 = 30$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

$\therefore 3! + 4! \neq 7!$

Question 3: If $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$, find x.

Answer 3:

$$\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$$

$$\Rightarrow \frac{1}{6!}+\frac{1}{7\times 6!}=\frac{x}{8\times 7\times 6!}$$

$$\Rightarrow \frac{1}{6!}\left(1+\frac{1}{7}\right)=\frac{x}{8\times 7\times 6!}$$

$$\Rightarrow 1+\frac{1}{7}=\frac{x}{8\times 7}$$

$$\Rightarrow \frac{8}{7}=\frac{x}{8\times 7}$$

$$\Rightarrow x=\frac{8\times 8\times 7}{7}$$

$$\therefore x=64$$

TRy

If $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$, find x.

$$\frac{15!}{12!\,3!}=\frac{15\cdot 14\cdot 13\cdot 12!}{12!\,3!}$$

$$=\frac{15\cdot 14\cdot 13\cdot \cancel{12!}}{\cancel{12!}\,3!}$$

$$=\frac{15\cdot 14\cdot 13}{3!}$$

$$=\frac{\overset{5}{\cancel{15}}\cdot \overset{7}{\cancel{14}}\cdot 13}{\cancel{3}\cdot\cancel{2}\cdot 1}$$

$$=5\cdot 7\cdot 13$$

$$=455$$

How to simplify???

$$\frac{(2m)!}{(2m+2)!} = \frac{(2m)!}{(2m+2)\cdot(2m+1)\cdot(2m)!}$$

$$= \frac{\cancel{(2m)!}}{(2m+2)\cdot(2m+1)\cdot\cancel{(2m)!}}$$

$$= \frac{1}{(2m+2)\cdot(2m+1)}$$

$$= \frac{1}{4m^2+4m+2m+2}$$

$$\frac{(2m)!}{(2m+2)!} = \frac{1}{4m^2+6m+2}$$

$$\frac{n!}{(n-2)!} = \frac{n\cdot(n-1)\cdot(n-2)!}{(n-2)!}$$

$$= \frac{n\cdot(n-1)\cdot\cancel{(n-2)!}}{\cancel{(n-2)!}}$$

$$= n(n-1)$$

$$\frac{n!}{(n-2)!} = n^2 - n$$

$$\frac{(n+1)!}{(n+3)!} = \frac{(n+1)!}{(n+3)\cdot(n+2)\cdot(n+1)!}$$

$$= \frac{\cancel{(n+1)!}}{(n+3)\cdot(n+2)\cdot\cancel{(n+1)!}}$$

$$= \frac{1}{(n+3)(n+2)}$$

$$= \frac{1}{n^2+3n+2n+6}$$

$$\frac{(n+1)!}{(n+3)!} = \frac{1}{n^2+5n+6}$$

Does order matter???

How many arrangements?

Tom & Jerry

Jerry & Tom

2 → **Arrangements**

2=2 x 1= 2!

Number of ways to arrange 3 different objects

ABC

A B C

A

B A C

B

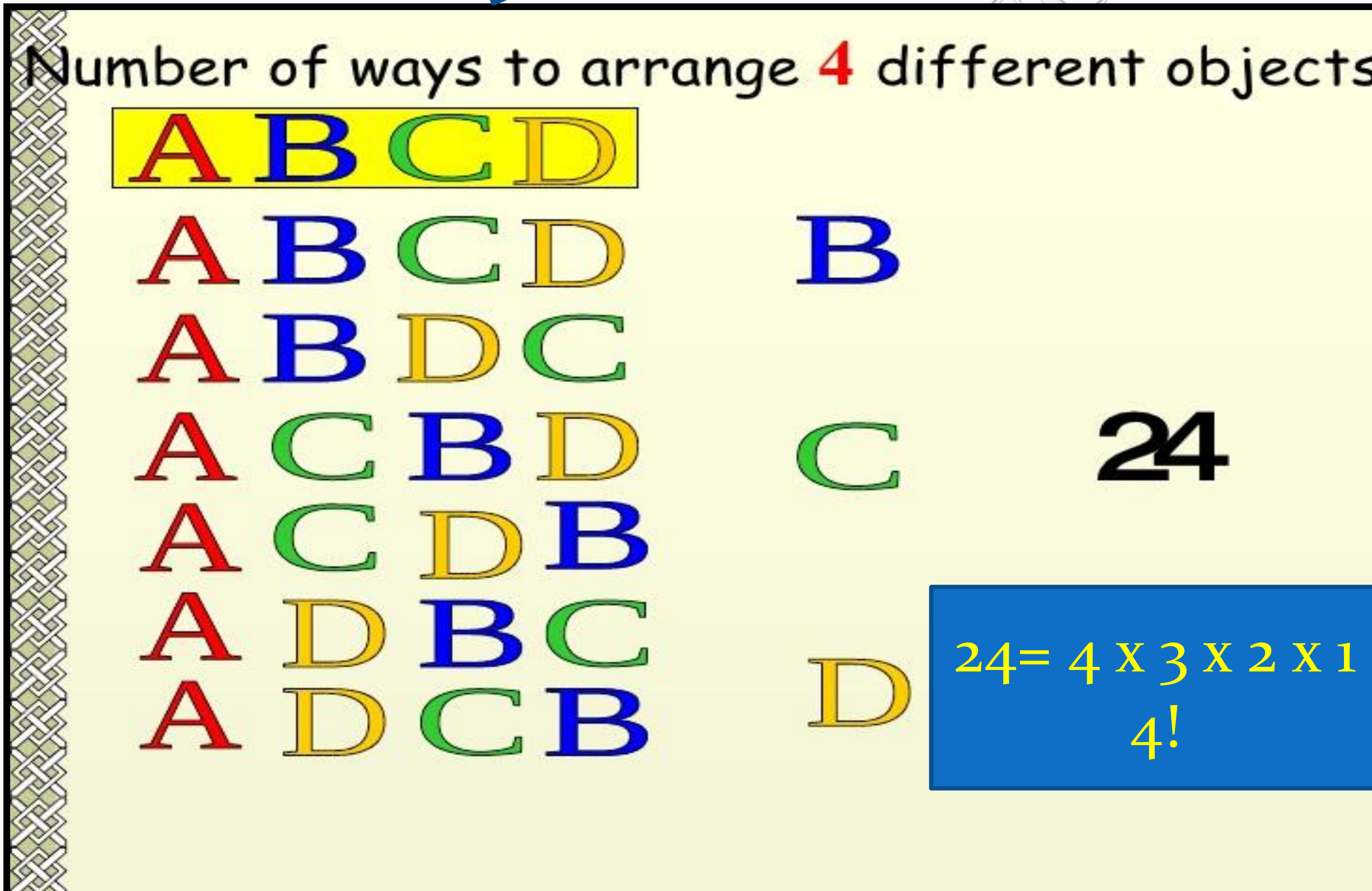
C A B

C

6 ways

6=3 x 2 x 1 =3!

How many???



24= 4 x 3 x 2 x 1

4!

No. of Objects → **No. of ways**

Factorial

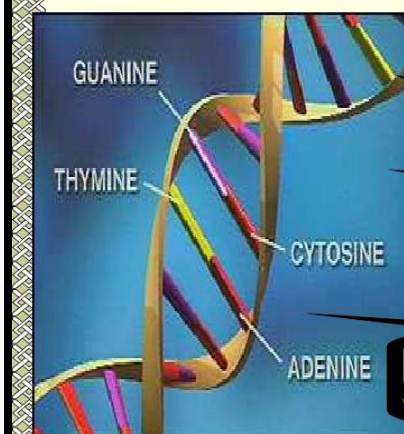
Number of objects	Number of ways		
1	1	1	= 1!
2	2	2 x 1	= 2!
3	6	3 x 2	= 3!
4	24	4 x 6 x 2 x 1	= 4!
5	120	5 x 24	= 5!
6		6 x 5 x 4 x 3 x 2 x 1	= 6!

The number of ways to arrange *n* objects = *n* !

1. To arrange 10 different objects = 10 !
2. To arrange digits 2, 5, 6, 8 = 4 !
3. To arrange 12 finalists = 12 !

4 different objects

12 different objects



DNA

23pairs of chromosomes

different ways to arrange

A Permutation is an arrangement in a definite order of number of objects taken some or all at a time

Permutations when all the objects are distinct

CONDITION
Objects do not repeat

Theorem 1

Number of permutations of **n** different objects taken **r** at a time is:

$$\boxed{{}^{n}P_{r}}$$

(Where $0 < r \leq n$)

$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

$^{n}P_{n} = n!$

$^{n}P_{1} = n$

$^{n}P_{0} = 1$

Number of ways to arrange 5 students from 8 students.

8 choices	7 choices	6 choices	5 choices	4 choices

8 x 7 x 6 x 5 x 4 = 6720

Using FPC

$$\text{No.of ways} = {}^{8}P_{5} = \frac{8!}{(8-5)!} = \frac{8!}{3!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 = 6720$$

Using Theorem 1

Example

How many different signals can be made by 3 flags from 4-flags of different colors?

Here **n= 4** and **r =3** as we need to make 3 flags out of 4 flags. Therefore...

$$ {}^{4}P_{3} = \frac{4!}{(4-3)!} = \frac{24}{(1)} = 24 $$

Exercise:7.3

Question 1:

How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Answer 1

3-digit numbers have to be formed using the digits 1 to 9.

Here, the order of the digits matters.

Therefore, there will be as many 3-digit numbers as there are permutations of 9 different digits taken 3 at a time.

Therefore, required number of 3-digit numbers $= {}^{9}P_{3} = 504$

$$ {}^{9}P_{3} = \frac{9!}{(9-3)!} = \frac{9!}{6!} $$

$$ \frac{9 \times 8 \times 7 \times 6!}{6!} = 9 \times 8 \times 7 = 504 $$

Exercise 7.3 continued

Question 2:

How many 4-digit numbers are there with no digit repeated?

T	H	T	O

Answer

The thousands place of the 4-digit number is to be filled with any of the digits from 1 to 9 as the digit 0 cannot be included.

Therefore, the number of ways in which **thousands place** can be filled is **9.**
Therefore , the number of ways of filling **hundreds, tens, and units place** = permutations of 9 different digits taken **3** at a time = ${}^{9}P_{3}$= **504**

$$ {}^{9}P_{3} = \frac{9!}{(9-3)!} = \frac{9!}{6!} $$

$$ \frac{9 \times 8 \times 7 \times 6!}{6!} = 9 \times 8 \times 7 = 504 $$

Thus, by multiplication principle, the required number of 4-digit numbers =9 × 504 = 4536

Question 5:
From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?

Answer 5:
From a committee of 8 persons, a chairman and a vice chairman are to be chosen in such a way that one person cannot hold more than one position. Here, the number of ways of choosing a chairman and a vice chairman is the permutation of **8** different objects taken 2 at a time = $^8P_2$

Thus, required number of ways = $^8P_2 = \frac{8!}{(8-2)!} = \frac{8!}{6!} = \frac{8 \times 7 \times 6!}{6!} = 8 \times 7 = 56$

Exercise 7.3. Qno.6

Find n if ${}^{n-1}P_3 : {}^{n}P_4 = 1:9$.

HOMEWORK
Exercise 7.3
Qs: 3 , 4 , 7 , 8

$${}^{n-1}P_3 : {}^{n}P_4 = 1:9$$

$$\Rightarrow \frac{{}^{n-1}P_3}{{}^{n}P_4} = \frac{1}{9}$$

$$\Rightarrow \frac{\left[\dfrac{(n-1)!}{(n-1-3)!}\right]}{\left[\dfrac{n!}{(n-4)!}\right]} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n \times (n-1)!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

$$\therefore n = 9$$

ASSIGNMENT

1.	**How many 4 letter words, with or without meaning can be formed out of the letters of the word WONDER ,if repetition of letters is not allowed?**
2.	Ten students participate in a debate. In how many ways can the first three prizes be won?
3.	In how many ways can three sports prizes be given to 20 boys when a boy receive any number of prizes?
4.	Evaluate $\frac{n!}{(n-r)!}$, when n = 6 , r= 2
5.	Find the total number of ways of answering 6 multiple choice questions, each question having 4 choices.
	ANSWERS: 1) ${}^{6}P_{4} = 360$. (2) ${}^{10}P_{3} = 720$ (3)20 x 20 x 20=8000 (4) 30 (5) 4 x 4 x 4 x 4 x 4 x4 $=4^{6}$

THANK YOU

Stay safe

Stay blessed